Proof $ln(x+1)$ can be represented by the Gaussian hypergeometric function denoted by $F(\alpha, \beta, \gamma; x)$*

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September 18, 2006

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\begin{aligned}
F(\alpha, \beta, \gamma; x) &:= 1 + \sum_{n=1}^{\infty} \frac{\alpha_n \beta_n}{n! \gamma_n} x^n \\
F(1, 1, 2; -x) &= 1 + \sum_{n=1}^{\infty} \frac{1_n 1_n}{n! 2_n} (-x)^n \\
\text{(note: } 1_n = n!) &= 1 + \sum_{n=1}^{\infty} \frac{n! 1_n (-x)^n}{2^n} \\
&= 1 + \sum_{n=1}^{\infty} \frac{(-x)^n}{n+1} \\
&= 1 + \sum_{n=1}^{\infty} \frac{(-1)^n (x)^n}{n+1} \\
xF(1, 1, 2; -x) &= x(1 + \sum_{n=1}^{\infty} \frac{(-1)^n (x)^n}{n+1}) \\
&= x + \sum_{n=1}^{\infty} \frac{(-1)^n x^{n+1}}{n+1} \\
\text{(shifting index)} &= x + \sum_{n=2}^{\infty} \frac{(-1)^{n-1} x^n}{n} \\
\text{[taylor series expansion for ln(1+x)]} &= x - \frac{x^2}{x} + \frac{x^3}{3} - \frac{x^4}{4} + ... \\
&= \ln(1+x) \quad \square
\end{aligned}
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*made with LaTeX